STUDIES ON THE STATICS AND KINEMATICS OF THE ATMOSPHERE IN THE UNITED STATES.

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VI. CERTAIN MATHEMATICAL FORMULÆ USEFUL IN METEOROLOGICAL DISCUSSIONS.

THE NEED OF A STANDARD SYSTEM OF FORMULÆ.

There is a large number of mathematical papers that have been written by meteorologists in the exposition of various theories, which must be thoroughly considered by students who seek to go beyond a descriptive statement of the problems into a close examination of the principles upon which the solutions rest. The question arose at an early stage in my study of comparative meteorology as to the form in which such mathematical discussions should be presented to the public. To traverse the entire range of treatises and explain them in detail was clearly impracticable; to adopt an abstract mathematical synopsis, such as is found in Carr's or Laska's synopsis of pure mathematics, was to put too great a strain upon readers who are not specialists in mathematical meteorology. Finally it seemed to me to be a fair compromise to take the following course: (1) reduce the important papers to one common standard notation, and (2) make an analysis of the result in a sufficiently expanded form to enable a good reader to follow the series of equations without difficulty. The only step required to transform the contents of the mathematical compendium as given in chapters 10 and 11 of the International Cloud Report into a complete treatise on analytic meteorology is to supply such transition precepts as are usually placed between the formulæ to aid the thought. It is, however, a distinct advantage for a working use of the formulæ, to one who has once become familiar with such problems, to dispense with these explanatory sentences, which only take up space. A ready reference to the standard equations under each subject is quickly appreciated by any one who uses these formulæ in a practical way, just as one would use a mathematical table in computing. It is my purpose to complete such a collection of formulæ, in addition to the tables contained in my report on Eclipse Meteorology and Allied Problems, Weather Bureau Bulletin I, 1902, by appropriate tables covering the subjects, spherical harmonics, thermodynamics, and the kinetic theory of gases, because these are indispensable in meteorological studies. I have taken the opportunity in this connection to present several original sets of formulæ, which have an advantage in their applications to meteorological problems, and it is my purpose to call attention to some of them in this paper.

THE GENERAL EQUATIONS OF MOTION.

The methods of deriving the general equations of motion on the rotating earth, as presented in Ferrel's paper, "The motions of fluids and solids on the earth's surface," or in the standard treatises of hydrodynamics, are so complicated as to discourage all who are not expert mathematicians from an examination of the solution. The fact that Ferrel did not evaluate the total differential of inertia $\frac{d\,(u,\,v,\,w)}{dt}$, introduced an error into the equations contained in his "Mechanics and general motions of the atmosphere," United States Coast Survey Report, 1875, Appendix 20; this was eliminated in his "Recent advances in meteorology," Annual Report of the Chief Signal Officer, 1885, Appendix 71. There are no doubt many ways of solving this problem, but the following is original, as expanded from Table 75, International Cloud Report, and it leaves little to be desired in respect of simplicity and completeness.

(1) THE POLAR EQUATIONS OF MOTION ON THE ROTATING EARTH.

Using the notation already adopted in Paper II of this series,[1] we write the primary equations of acceleration of motion referred to axes which have their origin at the center of a nonrotating earth, as follows:

The accelerations due to motion and to external forces are:

155.
$$-\frac{1}{\rho}\frac{\partial P}{\partial x} - \frac{\partial V}{\partial x} = \frac{du}{dt} - v\omega_3 + w\omega_2$$
$$-\frac{1}{\rho}\frac{\partial P}{\partial y} - \frac{\partial V}{\partial y} = \frac{dv}{dt} - w\omega_1 + u\omega_3$$
$$-\frac{1}{\rho}\frac{\partial P}{\partial z} - \frac{\partial V}{\partial z} = \frac{dw}{dt} - u\omega_2 + v\omega_1$$

where the angular velocities of motion for a point are

166.
$$\omega_1 = -\frac{v}{r} \qquad \omega_2 = +\frac{u}{r} \qquad \omega_3 = +\frac{v}{r\tan\theta}.$$

Compare diagram in my Report, page 498, or Basset, pages 13 and 14, noting the transformations of notation.

In case the earth rotates with the constant angular velocity n, carrying the fixed axes with it, the linear velocities (u, v, w) and the angular velocities $(\omega_1, \omega_2, \omega_3)$ are changed as follows, denoting these terms on the rotating earth with primes:

177.
$$u' = u \qquad v' = v + n\,r\sin\theta \qquad w' = w$$

178.
$$\omega'_1 = -\frac{v + n\,r\sin\theta}{r} \qquad \omega'_2 = +\frac{u}{r} \qquad \omega'_3 = +\frac{v + n\,r\sin\theta}{r\tan\theta}.$$

This is due to the fact that the rotation of the earth adds the velocity $n\,r\sin\theta = n\,\varpi$ to the eastward linear velocity, because ϖ is the perpendicular distance from the axis of rotation.

The differentials $\frac{du'}{dt}$, $\frac{dv'}{dt}$, $\frac{dw'}{dt}$ evaluate into,

179.
$$\frac{du'}{dt} = \frac{du}{dt}$$
$$\frac{dv'}{dt} = \frac{dv}{dt} + \frac{d\,(n\,r\sin\theta)}{dt} = \frac{dv}{dt} + u\,n\cos\theta + w\,n\sin\theta$$
$$\frac{dw'}{dt} = \frac{dw}{dt}$$

since $u = \frac{r d\theta}{dt}$ and $w = \frac{dr}{dt}$ by formulæ 153, page 497, of the International Cloud Report.

Substituting these values in the equations of motion for the rotating earth, which are the same as those of 155 with the letters all primed, and taking the equivalents of dx, dy, dz in polar cordinates from 153, we have:

180.
$$-\frac{1}{\rho}\frac{\partial P}{r\partial\theta} = \frac{du}{dt} - (v + n\,r\sin\theta)\frac{(v + n\,r\sin\theta)}{r\tan\theta} + w\frac{u}{r},$$
$$-\frac{1}{\rho}\frac{\partial P}{r\sin\theta\,\partial\lambda} = \frac{dv}{dt} + w\frac{(v + n\,r\sin\theta)}{r} + u\frac{(v + n\,r\sin\theta)}{r\tan\theta} + u\,n\cos\theta + w\,n\sin\theta,$$
$$-\frac{1}{\rho}\frac{\partial P}{\partial r} - g = \frac{dw}{dt} - u\frac{u}{r} - (v + n\,r\sin\theta)\frac{(v + n\,r\sin\theta)}{r}.$$

The external forces derived from the potential V are:

$$-\frac{\partial V}{\partial x} = 0, \qquad -\frac{\partial V}{\partial y} = 0, \qquad -\frac{\partial V}{\partial z} = -g.$$

Performing the algebraic work, these equations reduce to

[1] See Monthly Weather Review for February, 1902, Vol. XXX, p. 81.

181.
$$-\frac{1}{\rho}\frac{\partial P}{r\partial\theta} = \frac{du}{dt} - \frac{v^2\cot\theta + uw}{r} - 2n\cos\theta\,.\,v - r\,n^2\sin\theta\cos\theta\,,$$
$$-\frac{1}{\rho}\frac{\partial P}{r\sin\theta\partial\lambda} = \frac{dv}{dt} + \frac{uv\cot\theta + vw}{r} + 2n\cos\theta\,.\,u + 2n\sin\theta\,.\,w\,,$$
$$-\frac{1}{\rho}\frac{\partial P}{\partial r} - g = \frac{dw}{dt} - \frac{u^2+v^2}{r} - 2n\sin\theta\,.\,v - rn^2\sin^2\theta.$$

The successive terms are the inertia, the centrifugal forces, the deflecting force, and the forces which change the figure of the earth from a sphere into an elipsoid of revolution.

(2) THE CYLINDRICAL EQUATIONS OF MOTION ON THE ROTATING EARTH.

If the axis of rotation of the earth is taken as the axis of rotation in cylindrical coordinates, the tangential velocity $= v + n\,\varpi$; but if the axis of rotation is any radius of the earth extended above the surface, the tangential velocity becomes $= v + n\,\varpi\cos\theta$. Hence we have, in cylindrical coordinates,

182.
$$u' = u \qquad w'_1 = 0$$
$$v' = v + n\,\varpi\cos\theta \qquad w'_2 = 0$$
$$w' = w \qquad w'_3 = n\cos\theta + \frac{v}{\varpi}.$$

The differentials $\frac{du'}{dt}, \frac{dv'}{dt}, \frac{dw'}{dt}$ evaluate into,

183.
$$\frac{du'}{dt} = \frac{du}{dt}$$
$$\frac{dv'}{dt} = \frac{dv}{dt} + \frac{d\,(n\,\varpi\cos\theta)}{dt} = \frac{dv}{dt} + u\,n\cos\theta$$
$$\frac{dw'}{dt} = \frac{dw}{dt}$$

since $u = \frac{d\varpi}{dt}$, by formulæ 152, and $\cos\theta$ is a constant. Substituting these values in the equations of motion for the rotating earth, which are the same as those of 155, with the letters all primed, and taking the equivalents of dx, dy, dz, in cylindrical coordinates from 152, we have:

184.
$$-\frac{1}{\rho}\frac{\partial P}{\partial\varpi} = \frac{du}{dt} - (v + n\,\varpi\cos\theta)\left(n\cos\theta + \frac{v}{\varpi}\right)$$
$$-\frac{1}{\rho}\frac{\partial P}{\varpi\partial\varphi} = \frac{dv}{dt} + u\,n\cos\theta + u\left(n\cos\theta + \frac{v}{\varpi}\right)$$
$$-\frac{1}{\rho}\frac{\partial P}{\partial z} - g = \frac{dw}{dt}.$$

The external forces derived from the potential V are:
$$-\frac{\partial V}{\partial x} = 0, \qquad -\frac{\partial V}{\partial y} = 0, \qquad -\frac{\partial V}{\partial z} = -g.$$

Performing the algebraic work, these equations reduce to

185.
$$-\frac{1}{\rho}\frac{\partial P}{\partial\varpi} = \frac{du}{dt} - 2n\cos\theta\,.\,v - \frac{v^2}{\varpi} = \frac{du}{dt} - (2n\cos\theta + \nu_1)\,v$$
$$-\frac{1}{\rho}\frac{\partial P}{\varpi\partial\varphi} = \frac{dv}{dt} + 2n\cos\theta\,.\,u + \frac{uv}{\varpi} = \frac{dv}{dt} + (2n\cos\theta + \nu_1)\,u$$
$$-\frac{1}{\rho}\frac{\partial P}{\partial r} - g = \frac{dw}{dt}$$

where the term $+\,\varpi\,n^2\cos^2\theta$ is neglected in the first equation, and $\nu_1 = \frac{v}{\varpi}$ the relative angular velocity.

The successive terms are the inertia, the deflecting force, and the centrifugal forces.

REMARKS ON THE SEVERAL TERMS IN THE GENERAL EQUATIONS OF MOTION.

It is customary to add to the terms developed in a frictionless medium, a term expressing the retardation of acceleration due to friction, either in Ferrel's form $+\,k\,(u, v, w)$, which is proportional to the velocity and expresses a sliding friction, or in Oberbeck's form $\frac{k}{\rho}\,\Delta^2\,(u, v, w)$, which expresses a retardation proportional to the turbulent internal resistances of a mixing fluid. This function is hard to evaluate on account of the uncertainty which attaches to the invisible internal motions, and to the effect of discontinuous surfaces separating different velocities and temperatures. Near the ground turbulent motions and large coefficients of friction up to about 300–500 meters are required; above this level and especially in the higher strata the coefficient of friction is very small.

The inertia terms $\frac{d\,(u, v, w)}{dt}$ disappear in steady motion, and they are small in slow changes of velocities. There are, however, cases in which inertia may amount to a considerable quantity, as where a tornado, in passing along its path, sucks in new masses of air, and transforms them suddenly from rest into excessively rapid motion. Also, when the cyclonic vortex raises masses of air from strata having slow motion into strata of rapid velocities; but especially where countercurrents meet, and the stream lines are bent and reflexed in their direction.

These two terms, friction and inertia, act in the path of motion and they directly affect the quantity of kinetic energy possessed by the elementary masses. All forces which act at right angles to the path, such as the centrifugal and the deflecting forces, do not change the momentum, but they do alter the direction of the path. Hence, in integrating for the kinetic energy in an orbit, or in a circuit, the centrifugal and the deflecting forces drop out of the equations, but they must be retained when discussing the angle that the stream line makes with the isobars, which angle expresses the influence of the velocity potential function on the motion. The following integration of the general equations will establish these propositions.

INTEGRATION OF THE GENERAL EQUATIONS OF MOTION IN POLAR COORDINATES.

Make the following substitutions in 181:

197.
$$\frac{v}{r} = \nu\sin\theta$$
$$\frac{v^2\cot\theta}{r} = v\cos\theta\,.\,\nu$$
$$\frac{u\,v\cot\theta}{r} = u\cos\theta\,.\,\nu$$

and neglect the terms in n^2, which are very small, with the result that,

200.
$$-\frac{1}{\rho}\frac{\partial P}{\partial x} = \frac{du}{dt} - \cos\theta\,(2n + \nu)\,v + \frac{uw}{r}$$
$$-\frac{1}{\rho}\frac{\partial P}{\partial y} = \frac{dv}{dt} + \cos\theta\,(2n + \nu)\,u + \sin\theta\,(2n + \nu)\,w$$
$$-\frac{1}{\rho}\frac{\partial P}{\partial z} = \frac{dw}{dt} - \sin\theta\,(2n + \nu)\,v - \frac{u^2}{r} + g.$$

Now multiply these equations respectively by dx, dy, dz, and remember that $v\partial x = u\partial y$, $w\partial x = u\partial z$, $w\partial y = v\partial z$; take the sum of the partial differentials, the result being,

203. $$-\frac{\partial P}{\rho} = u\partial u + v\partial v + w\partial w + g\partial z.$$

The integral of this is,

$$\int -\frac{\partial P}{\rho} = \tfrac{1}{2}(u^2 + v^2 + w^2) + gz + \text{const.} = \tfrac{1}{2}q^2 + gz + \text{const.}$$

This is the fundamental equation of steady motion found in all treatises on hydrodynamics; its discussion is carried on in Table 81, International Cloud Report. The centrifugal and the deflecting forces have disappeared, and the integral is equivalent to the kinetic energy, $\frac{1}{2}q^2$, plus the external force due to the acceleration of gravitation. An arbitrary term may be added to express the frictional retardation.

If the integration is between two points of a fluid that has the same density throughout, the term $\int -\frac{\partial P}{\rho} = -\frac{P}{\rho}$ simply. Such lines of homogeneous integration may be found by observing the surfaces of equal density in the atmosphere, or, a mean density between two points may be assumed in place of the existing variable density. If the velocity term $\frac{1}{2}q^2$ is neglected, we obtain $dP = -g\rho dz$, and this is the simple form from which the usual hypsometric formulæ for barometric reductions are derived. Compare formulæ 54, Table 66, p. 490.

It is noted, however, that the usual method employed in static barometric reductions is incomplete, and that the velocity term $\frac{1}{2}(q^2 - q_0^2)$, where q, q_0 are the observed velocities at the two points limiting the path of integration, has been omitted.

If the integration is continued in any closed circuit the gravity term disappears from the equation, and the velocity terms alone remain. This line integral (C.) measures the work done in moving the unit mass once around the circuit, while $A = \frac{dC}{dt}$ is the rate of doing the work, or the activity. From this point of view the circulation of the atmosphere may be treated by the ordinary theory of the line integral. It is more convenient to observe the velocities than the pressures and densities around a circuit, in the present state of meteorology.

EXPRESSIONS FOR THE GRADIENTS OF PRESSURE.

If we take the formulæ for acceleration, Cloud Report, page 499,

155. $$\dot{u}_1 = -\frac{1}{\rho}\frac{\partial P}{\partial x} - \frac{\partial V}{\partial x}$$

$$\dot{v}_1 = -\frac{1}{\rho}\frac{\partial P}{\partial y} - \frac{\partial V}{\partial y}$$

$$\dot{w}_1 = -\frac{1}{\rho}\frac{\partial P}{\partial z} - \frac{\partial V}{\partial z}$$

we can write for the gradient,[1]

501. $$G_x = -\frac{\partial P}{\partial x} - \rho\frac{\partial V}{\partial x} = \rho\,\dot{u}_1$$

$$G_y = -\frac{\partial P}{\partial y} - \rho\frac{\partial V}{\partial y} = \rho\,\dot{v}_1$$

$$G_z = -\frac{\partial P}{\partial z} - \rho\frac{\partial V}{\partial z} = \rho\,\dot{w}_1.$$

The gradient terms, $-\rho\frac{\partial V}{\partial x}$ in latitude, and $-\rho\frac{\partial V}{\partial y}$ in longitude, are small terms, while $-\rho\frac{\partial V}{\partial z}$ is the principal term, and these are due to the attraction of the earth upon the atmosphere. The terms $-\frac{\partial P}{\partial x}$ in latitude, $-\frac{\partial P}{\partial y}$ in longitude, and $-\frac{\partial P}{\partial z}$ in altitude are the gradients due to the thermal disturbance of the isobaric surfaces, the first two being the gradient terms producing the horizontal flow of the atmosphere, and the last one the term which causes the up and down movement of the atmosphere by the variations of the normal buoyancy from that of stable equilibrium as controlled by the static terms in the potential function for external force V.

[1] The series of equations beginning with 501 may be considered as an extension of the system given in the International Cloud Report, which ends on page 603.

It is next important to evaluate the gradient terms for use in practical meteorology. There are many ways of doing this, as is indicated by the collection of formulæ in Table 65, page 489, of the International Cloud Report. There is a generally accepted convention which is adopted as the basis for the practical measures of gradients by the mercurial barometer.

Thus, the difference of barometric pressure, G, at two points which are 111 111 meters apart in a horizontal direction, is taken as the standard for reductions.

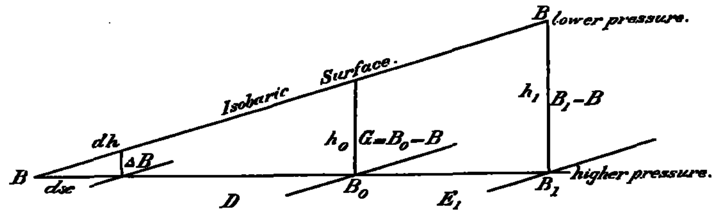


FIG. 21.—Vertical section through the atmosphere.

In fig. 21, which shows a vertical section through the atmosphere, let

D = 111 111 meters = 1° on surface of the earth,
dx = 1 meter,
E = any distance between given points of observation.

Then,

502. $$\frac{B_1 - B}{E_1} = \frac{B_0 - B}{D} = \frac{G}{D} = \frac{G}{111\,111} = \frac{\Delta B}{dx};$$

$$G = (B_1 - B)\frac{D}{E_1}.$$

It is necessary first to find G from the observed values of B at two stations at the distance E_1 from each other.

EVALUATION OF THE COEFFICIENT $\frac{dh}{dB}$ AND OTHER TERMS.

If the change in elevation of the isobaric surface is as follows: h_1 at distance E_1, h_0 at distance D, dh at distance dx, then,

503. $$g\frac{h_1}{E_1} = g\frac{h_0}{D} = g\frac{dh}{dx} = g\tan a = gb \text{ measures the acceleration.}$$

Also by the law of falling bodies, $v = \sqrt{2gh}$ for the velocity.

I. We have,

504. $\frac{dh}{dx} = \frac{h_0}{D} = \frac{h_1}{E_1} = \frac{l}{E_l}$ for the top of the homogeneous atmosphere.

505. $\frac{dB}{dx} = \frac{B_0 - B}{D} = \frac{G}{D} = \frac{G}{111\,111} = \frac{\partial P}{\partial x}\cdot\frac{1}{g_0\rho_m}$, from 24, p. 487 of the Cloud Report.

Hence,

506. $$\frac{dh}{dB} = \frac{h_0}{B_0 - B} = \frac{l}{E_l}\,\frac{D}{G} = \frac{l}{E_l}\,\frac{E_1}{B_1 - B} = \frac{l}{E_l}\,\frac{E_l}{B_l - B} = \frac{l}{B_l} = \frac{RT}{B_l}$$

$$\text{since } \frac{D}{G} = \frac{E_1}{B_1 - B} = \frac{E_l}{B_l - B} = \frac{E_l}{B_l},$$

because B at the top of the homogeneous column l is negligible compared with B_l at the bottom of it. B_l is in this connection the barometric pressure at the surface, and $B_l = B_n = 0.760$ meter.

II. We have by 50, page 490, for the standard weight of the atmosphere,

507. $$p_o = \sigma h = \sigma_o l = \sigma_m B_n. \quad \text{Hence,}$$

508. $$h = \frac{\sigma_o}{\sigma} l = \frac{\sigma_m}{\sigma} B_n. \quad \text{That is, } h = l \text{ for } \sigma = \sigma_o. \quad \text{Hence,}$$

509. $$h = l = \frac{\sigma_m}{\sigma_o} B_n. \quad \text{Therefore,}$$

510. $$s = \frac{\sigma_m}{\sigma_o} = \frac{l}{B_n} = \frac{RT}{B_n} = \frac{13,595.8}{1.29305} = \frac{7,991.04}{0.760} = 10,514.5.$$

511. $$dh = \frac{\sigma_m}{\sigma_o} dB = \frac{RT}{B_n} dB = 10,514.5\, dB = s\, dB.$$

512. $$\frac{dh}{dx} = 10,514.5 \frac{B_o - B}{D} = 10,514.5 \frac{G}{111\,111} = 0.09463G.$$

III. Let Γ = the gradient force per meter; that is, for $dx = 1$.

513. $\Gamma = \Delta P = g_o \rho_m \Delta B$ in terms of the units of force P.

514. $\Gamma = \Delta p = \sigma_m \Delta B$ in terms of the units of weight p.

The gradient force changes with the temperature.

Let Γ_o = the gradient force for $T_o = 273°$ C. and $B_n = 0.760$ meter.

Γ = the gradient force for T and B.

515. $$\Gamma = \Gamma_o \frac{T}{T_o} \frac{B_n}{B}.$$

IV. To evaluate $-\frac{1}{\rho}\frac{\partial P}{\partial x}$ and $-\frac{1}{\rho}\frac{\partial p}{\partial x}$:

516. We have $P_o = g_o \rho_m B_n$; and hence,

517. $$-\frac{1}{\rho}\frac{\partial P}{\partial x} = -\frac{1}{\rho} g_o \rho_m \frac{\partial B}{\partial x} = -\frac{1}{\rho}\frac{g_o \rho_m}{111\,111} G = -\frac{0.0012G}{\rho}.$$
(G is in meters.)

518. Also, we have $p_o = \sigma_m B_n$; and hence,

519. $$-\frac{1}{\rho}\frac{\partial p}{\partial x} = -\frac{1}{\rho}\sigma_m \frac{\partial B}{\partial x} = -\frac{1}{\rho}\frac{\sigma_m}{111\,111} G = 0.12237G.$$
(G is in meters.)

Numerous other evaluations of $-\frac{1}{\rho}\frac{\partial P}{\partial x}$ are given in Table 65, p. 489, of the Cloud Report.

EVALUATION OF THE GRADIENTS IN POLAR COORDINATES.

Since the angular velocity of the rotating earth is $n \sin\theta = \frac{v'}{r}$, where v' is the absolute eastward velocity, and $r = 6,370,191 + h$ meters, we have $n = 0.00007292$, and also $n \cos\theta = \frac{v' \cot\theta}{r}$, in which r can be taken practically equal to R. The general polar equations of motion become, by substituting these values in 181,

194. $$-\frac{1}{\rho}\frac{\partial P}{\partial x} = \frac{du}{dt} - \frac{\cot\theta}{r}(2v' + v)v + \frac{uw}{r}$$
$$-\frac{1}{\rho}\frac{\partial P}{\partial y} = \frac{dv}{dt} + \frac{\cot\theta}{r}(2v' + v)u + (2v' + v)\frac{w}{r}$$
$$-\frac{1}{\rho}\frac{\partial P}{\partial z} = \frac{dw}{dt} - \frac{1}{r}(2v' + v)v - \frac{u^2}{r} + g.$$

The terms in n^2 which give the figure to the rotating earth have been omitted, and the inertia terms become equal to zero for steady motions of the atmosphere; also, for all except computations of great precision the terms in w can be neglected.

To evaluate the acceleration $\frac{1}{\rho}\frac{\partial P}{\partial x}$, we have, first, from 47,

47a. $$\frac{1}{\rho} = \frac{1}{\rho_o}\frac{P_o}{P}\frac{T}{T_o} = \frac{1}{\rho}\frac{P_o}{P}(1 + at)$$
$$= \frac{1}{\rho_o}\frac{B_n}{B}\frac{T}{T_o}\frac{1}{n_1}, \text{ for variations of gravity, since } g = g_o n_1,$$
$$= \frac{1}{\rho_o}\frac{760}{273}\frac{T}{B} \quad \text{for constant gravity and } B \text{ in mm.}$$

From the formulae on page 489,

47b. $$\frac{1}{\rho}\frac{\partial P}{\partial x} = \frac{1}{\rho} g_o \rho_m \frac{dB}{dx}, \text{ and since } \frac{dB}{dx} = \frac{G_x}{D} = \frac{G_x}{111\,111} \text{ in meters}$$
$$= \frac{1}{\rho}\frac{g_o \rho_m}{111\,111} G_x \text{ for the gradient measured in meters}$$
$$= \frac{1}{\rho}\frac{g_o \rho_m}{111\,111\,111} G_x \text{ for the gradient } G_x = B_o - B \text{ in mm}$$
$$= \frac{1}{\rho_o}\frac{B_n}{T_o}\frac{T}{B}\frac{g_o \rho_m}{D} G_x$$

520. $$= \frac{13.5958}{0.00129305} \times \frac{760}{273} \times \frac{9.806}{111\,111\,111} \times \frac{T}{B} \times G_x$$

521. $$= 0.0025833 \frac{T}{B} G_x$$

522. $$= \frac{\cot\theta}{r}(2v' + v)v, \text{ by equation 194.} \quad \text{Hence,}$$

523. $$G_x = +387.102 \frac{B}{T}\frac{\cot\theta}{r}(2v' + v)v,$$

and similarly,

$$G_y = -387.102 \frac{B}{T}\frac{\cot\theta}{r}(2v' + v)u,$$
$$G_z = +387.102 \frac{B}{T}\left[\frac{1}{r}(2v' + v)v + \frac{u^2}{r} - g\right].$$

Since v' is a function of θ, that is, $v' = nr\sin\theta$, these terms can be computed by simple tables, such as those in Tables 104, 105, 106, of the International Cloud Report, where some of the terms are evaluated. By expressing the variation of $387.102 \times \frac{B}{T}$ in a table with B and T as the arguments, the several products can be quickly computed.

Examples:

I. For $B = 700$ mm. and $T = 260°$ C., $\frac{B}{T} = 2.6923$;

For $\theta = 30°$ north polar distance, $2v' = 464.5$ meters per second;

For $v = 40$ meters per second, $(2v' + v)v = 20,180$.

Hence, $G_x = 387.102 \frac{B}{T}\frac{\cot\theta}{R}(2v' + v)v = 5.71$ millimeters per 111 111 meters.

II. This latter has been computed from the tables as follows:

$$378.102 \frac{B}{T} = 1,042.2;$$
$$\frac{v'}{R}\cot\theta \,.\, 2v = 0.005052, \text{ by Table 104;}$$
$$\frac{\cot\theta}{R} \,.\, v \,.\, v = 0.000435, \text{ by Table 106.}$$

The sum of these is $\frac{\cot\theta}{R}(2v' + v)v = 0.005487$. Hence the product, $G_x = 1,042.2 \times 0.005487 = 5.71$ millimeters per 111 111 meters. Similarly, the gradients G_y and G_z can be computed.

For these values of B and T, we find in other examples,

$$\left.\begin{matrix}\theta = 40° \\ v = 45°\end{matrix}\right\} G_x = 6.89 \quad \left.\begin{matrix}\theta = 50° \\ v = 50°\end{matrix}\right\} G_x = 5.23 \quad \left.\begin{matrix}\theta = 60° \\ v = 55°\end{matrix}\right\} G_x = 4.47.$$

We are at last in a position to examine the system of gradients in the United States on the 10,000-foot plane and on the 3,500-foot plane. For we have obtained by the nephoscope and theodolite observations as given in the Cloud Report a large number of corresponding values of u and v, which enter these equations. The values of B and T on these planes have been carefully determined for each month, and also the gradients by which such values can be determined at any time. This will enable us to discuss the effect of friction at these planes, by means of the residuals which occur between the values of G as found by these formulæ and those read off from the charts of isobars contained in the Barometry Report of 1900–1901.

Furthermore, our Weather Bureau stations will soon be provided with suitable tables for computing pressures on the 3,500-foot and the 10,000-foot planes, and this will give daily configurations of isobars on these two levels. If, in addition, we had measures of the velocity of the clouds, $q\ (u, v, w)$, above each station by means of nephoscopic observations, it would enable us to make complete dynamic computations of the forces acting in cyclones and anticyclones, as is seen by an inspection of the formulæ.

Since the tabular computations are constructed for average conditions, it is of the utmost importance that *check observations* be made on these two planes in order to control these dynamic discussions and make them more perfect. Such observations can be made by balloon ascensions up to 2 or 3 miles, or by kite ascensions up to 10,000 feet, or by certain computations on cloud observations.

It seems to me very clear that a series of suitable research explorations would soon result in placing our dynamic meteorology upon a satisfactory scientific basis, and put an end to the fruitless speculations which have done so little to advance our knowledge of the laws of the atmospheric motions.

THE EQUATION OF CONTINUITY, AND SOME DERIVED RELATIONS.

1. The equation of continuity can be found as follows:

Consider a cylinder of the height z and radius ϖ, into which air of the density ρ streams equally from all sides with the velocity $-u$, since the direction is negative.

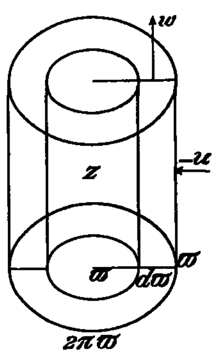


FIG. 22.—Illustrating the formation of the equation of continuity.

The amount of instreaming air in the unit of time is

$$-2\pi\varpi\, z\, u\rho.$$

If the air is incompressible, then there will stream into a cylinder, whose radius is smaller by $d\varpi$, the amount $-2\pi\,(\varpi - d\varpi)\, zu\rho$, and at the same time there will escape upward between these two cylinders the amount $2\pi\varpi\ d\varpi\ w\rho$. Hence

524. $$-2\pi\varpi\, z\, u\rho + 2\pi\,(\varpi - d\varpi)\, z\, u\rho = -2\pi d\varpi\, z u\rho = 2\pi\varpi d\varpi\, w\rho.$$

Integrating along the entire radius from 0 to ϖ, we have,

$$-2\pi\rho\int_0^{\varpi} zu\,d\varpi = 2\pi\rho\int_0^{\varpi}\varpi\,d\varpi\ w.$$

Therefore, $-zu\varpi = \frac{1}{2}\varpi^2 w$, and the equation of continuity becomes

487. $$-2uz = \varpi w.$$

This applies to pure vortex motion, and it finds some examples in the atmosphere, such as in tornadoes, in many hurricanes, and in some highly developed cyclones.

It may be remarked that in treating of the general circulation of the atmosphere, the application of the pure vortex law $\varpi v =$ constant, has failed to give correct results, for example in the writings of Ferrel, von Helmholtz, Oberbeck, Sprung, and others. This leads to the theory of contracting rings on the earth with progressive motion towards the poles, or expanding rings with progression towards the equator. While the law of the sum of the momenta $\Sigma\, mv = 0$ must prevail, the rings are in nature broken up into such complex stream lines as to render integration by the simple vortex law too rough and ready a method. We must, therefore, study the theory of typical stream lines, before attempting any general integration for the entire circulation.

The following derived relations are convenient:

2. Since $-2uz = \varpi w$, we have $-u = \frac{w}{2z}\varpi$

$$= \frac{c}{2}\varpi, \text{ if } w = cz.$$

3. For z and $-u$ both constant, we have $\varpi z u = -$ const. $= \psi$.

Hence, $z = -\frac{\text{const.}}{u\varpi} = -\left(\frac{\text{const.}}{c/2}\right)\frac{1}{\varpi^2}$, and by differentiation,

$$dz = +\frac{\text{const.}}{c/2}\cdot\frac{2\varpi d\varpi}{\varpi^4} = -\varpi^2 z\frac{2d\varpi}{\varpi^3}$$

$$= -2z\frac{d\varpi}{\varpi}.\quad \text{Therefore,}$$

$$\frac{dz}{z} = -2\frac{d\varpi}{\varpi};\ \text{also, } \varpi\frac{dz}{dt} = -2z\frac{d\varpi}{dt}.$$

4. These give the form for the current function ψ, and the velocity potential φ, in two cases.

488. I. $\psi = u\varpi z = -\frac{c}{2}\varpi^2 z = \varphi z.$ II. $\psi = -cz.$

489. $\varphi = -\frac{c}{2}\varpi^2.$ $\varphi = -c.$

$a = \frac{\lambda}{k-c}.$ $a = \frac{\lambda}{k}.$

5. If the current function is modified through a deflecting force and also through friction, then the equation of motion has two solutions, so that the roots of

$$u\,\frac{\partial v}{\partial\varpi} + \frac{uv}{\varpi} + \lambda u + kv = 0$$

are, by 438,

1. $$u = -\frac{c}{2}\varpi.$$

$$v = +\frac{\lambda}{k-c}\,\frac{c}{2}\,\varpi = -\frac{\lambda}{k-c}\,u.$$

2. $$u = -\frac{c}{\varpi}.$$

$$v = +\frac{\lambda}{k}\,\frac{c}{2} = -\frac{\lambda}{k}\,u.$$

In obtaining the velocities of the rotation, v, we can modify the current function, as follows, namely, multiply by

$$a = \frac{\lambda}{k-c},\ \text{and}\ a = \frac{\lambda}{k}\ \text{in the two cases.}$$

6. Hence, by using Stokes's current functions, we find for the velocities u, v, w in the two cases,

490. Case I.
$$u = +\frac{1}{\varpi}\frac{\partial\psi}{\partial z} = \frac{\varphi}{\varpi} = -\frac{c}{2}\varpi.$$
$$v = +\frac{a\psi}{\varpi} = -\frac{\lambda z}{k-c}\,\frac{c}{2}\,\varpi = +\frac{\lambda}{k-c}z\,u.$$
$$w = -\frac{1}{\varpi}\frac{\partial\psi}{\partial\varpi} = +cz = -\frac{2u}{\varpi}z = -\frac{2v}{\varpi}\,\frac{k-c}{\lambda}.$$

Case II.
$$u = +\frac{1}{\varpi}\frac{\partial\psi}{\partial z} = \frac{\varphi}{\varpi} = -\frac{c}{\varpi}.$$
$$v = +\frac{a\psi}{\varpi} = -\frac{\lambda}{k}\frac{c}{\varpi}z.$$
$$w = -\frac{1}{\varpi}\frac{\partial\psi}{\partial\varpi} = 0.$$

7. In unconstrained motion the vortex law of preservation of areas is

$$v\varpi = \frac{c}{2}\varpi^2 z = 2g\frac{\varpi z}{v} = \varpi u z = \text{constant, by 307.}$$

$$w\pi\varpi^2 = \text{const., by 308, introducing the value } v^2 = 2gz.$$

This vortex law when not modified by deflection and friction becomes,

492. Case I.
$$v\varpi = \frac{\lambda}{k-c}\,\frac{c}{2}\,\varpi^2 z = -\text{const.}$$

Case II.
$$v\varpi = \frac{\lambda}{k}cz = -\text{const.}$$

8. The inclination of the stream line to the isobars is,

491. Case I.
$$\cot i = +\frac{\lambda}{k-c}z.$$

Case II.
$$\cot i = \frac{\lambda}{k}z.$$

9. The equation of continuity (163) is satisfied by these following values:

493.
$$\frac{\partial u}{\partial\varpi} + \frac{u}{\varpi} + \frac{\partial w}{\partial z} = -\frac{c}{2} - \frac{c}{2} + c = 0.$$

10. The equation for gradient has a term to express the unevaluated variation due to temperature effects, $f(t_x)$, and it becomes, for the radial component,

494.
$$-\frac{\mu}{\rho}G_x = -\frac{1}{\rho}\frac{\partial P}{\partial x}$$
$$= -\frac{c}{2}\varpi\left[k-c+\lambda\frac{\lambda z}{k-c} + \frac{c}{2}\left(\frac{\lambda z}{k-c}\right)^2\right] + f(t_x)$$
$$= -\frac{c}{2}\varpi\left[k-c+\lambda\cot i + \frac{c}{2}\cot^2 i\right] + f(t_x).$$

11. The total velocity is

495.
$$q^2 = u^2 + v^2 + w^2 = \frac{u^2}{\sin^2 i} + w^2 = u^2\left(\frac{1}{\sin^2 i} + \frac{4z^2}{\varpi^2}\right) + f(t_x).$$

12. The variation of pressure can be expressed by

496.
$$\log P_0 - \log P = \frac{(q^2 - q_0^2) + 2g(z - z_0)}{360862\,(1 + at)} + f_1(k) + f_2(t).$$

These formulæ are all collected in Table 121, page 602, of my International Cloud Report.

This system of formula applies directly only to the pure vortex motions that satisfy the assumed current function and velocity potential. The components u, v, w, are so simply interrelated that it is usually possible to make enough observations of some sort from which to derive all the other vortex relations. Applications of them were made in the International Cloud Report to two cases; (1) The waterspout observed off Cottage City, Marthas Vineyard, Mass., August 19, 1896, on page 633; upon this important formation, a fuller report will be published. (2) The average velocities in a cyclone from the data in Table 126, as given on page 629 of the International Cloud Report. The outcome of these computations is to show that the natural stream lines of the atmosphere conform on the average to these formulæ. There are, however, wide divergences of such a type as to indicate that the pure vortex motion is seriously modified by several conflicting forces, and that the true problems for the meteorologist consist in discovering the nature and amount of these deviations of the currents of the atmosphere from the simple laws. This is in fact a task of great difficulty, but it has now become evident what should be the course of scientific development for meteorology. There is little use in a further discussion of the general theorems at the present time, but there is great need of procuring the right kind of observations for use in such problems. The Weather Bureau has accordingly been engaged in such a reconstruction of its data as will contribute to the solution of these problems for the United States. We have already published a large number of nephoscope velocities for the eastern half of the country; the velocities of the upper currents for the West Indies have been determined for about three years, July 1899–July 1902, and their computation will be commenced at once; similar nephoscope observations will be undertaken for the Rocky Mountain and Pacific districts, beginning about July 1902. Our barometric observations have been thoroughly reduced for the years 1873 to the present time, and the tables necessary for reductions to the three reference planes are in hand for the construction of daily maps at three levels, containing the system of isobars corresponding with them. It will be necessary to revise the temperature and vapor tension observations and reduce them to homogeneous systems before our data will be complete for the application of the theoretical equations to the observational data. It is desirable to put an end to general mathematical speculation in meteorology, and to substitute for it definite comparisons between observations and computations together with dependent solutions for the outstanding unknown quantities.

THE PROBLEMS OF THE AQUEOUS VAPOR CONTENTS OF THE ATMOSPHERE.

I shall allow myself only a few remarks regarding the methods which were used in my report for the discussion of the various complicated problems that concern the aqueous vapor contents of the atmosphere, because the details are too complex for a brief summary like this, and also because the work was given in such an extended form as to enable students to follow it without difficulty. There are, however, a few leading ideas to which attention may be especially directed, as they serve for an introduction to the subject in general.

There is collected in the International Cloud Report, Table 64, "Fundamental constants," a series of elementary constants in the English and metric systems, with the logarithms of the constants, and also a set of elementary formulæ which are most useful in meteorological studies. They cover nearly all the simple relations which constantly recur in manifold forms in the treatises and papers on meteorological subjects, and by transformation and combination a multitude of different relations can be readily obtained. Tables 63, 64, and 65 supply the basis for much descriptive matter commonly found in treatises, in so compact and accurate a form as to quite supersede the lengthy statements with which the same laws are usually presented, and this is a great convenience for the student and computer. Those who will take the trouble to become familiar with these tables will find much saving of time in general work, and also they will be guarded from such errors of thought and statement as are likely to occur from not having these formulæ in mind, or accessible for convenient reference.

In treating the vapor problems I have referred all the formulæ to the ratio $\frac{e}{B}$, vapor tension divided by barometric pressure, as the most convenient and accurate argument for combination with another argument, as the height h, the temperature T, or the pressure B. The Table 67 summarizes the formulæ for the hypsometric reductions, and they are more fully explained in the forthcoming Barometry Report. The general idea is that having found the ratio $\frac{e_o}{B_o}$ at the base of a column, the application of Hann's law for the diminution of the vapor pressure with the height gives the most accurate average law for computing the integral of the vapor tension throughout the entire column. A small secondary term can be added whenever our knowledge of the facts justifies such an increased degree of accuracy, though it is usually of little importance, especially for a series of observations where mean results are required.

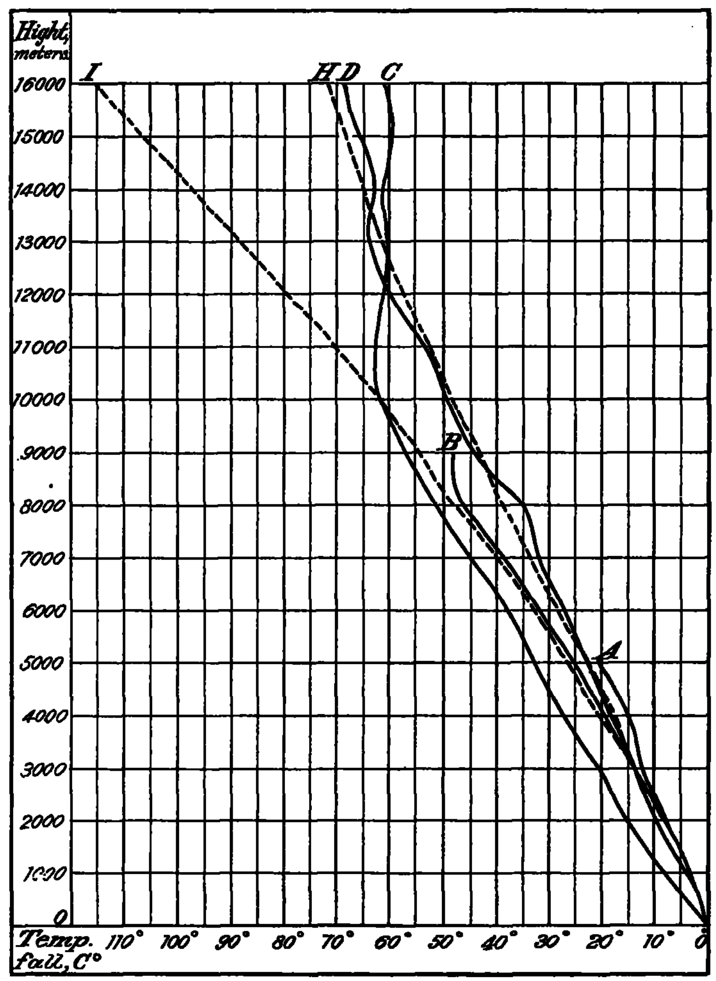
In the development of the α, β, γ, δ stages of the adiabatic thermodynamic formulae, the ratio $\frac{e}{B}$ is made the primary argument by the series of transformations given on Table 72. These formulæ are reduced to numerical tables, 94–102, and their accuracy is tested by comparing directly with the Hertzian logarithmic formulæ, as given in the examples of Table 108. Their use involves a series of solutions by trials, which though laborious, yet lead to perfectly rigorous results, and after a little practise it becomes quite easy to obtain the true trial values without much difficulty. The graphical diagrams of Hertz give only approximate values, because they throw out the vapor tension term in the critical places and thus render inaccurate the very problems they were designed to discuss. Special applications were made to finding the gradients of pressure, temperature, and vapor tension in the α, β, γ, δ stages, and the results are found in Tables 147 for metric measures, and in Tables 153 for English measures.

TABLE 21.—*Comparison of several determinations of the total temperature change from the surface to high levels.*

	A.	*B.*	*C.*	*D.*	*E.*	*F.*	*G.*	*H.*	*I.*
16000	....	....	—60.4	—68.3	....	......	......	—71.1	—115.0*
15000	....	....	—59.1	—66.0	....	......	......	—68.0	—106.0*
14000	....	....	—60.9	—62.5	....	......	......	—64.7	— 97.0*
13000	....	....	—60.1	—63.5	....	......	......	—61.0	— 88.0*
12000	....	....	—61.0	—60.3	....	......	......	—57.0	— 79.0*
11000	....	....	—62.8	—52.7	....	......	......	—52.6	— 70.0*
10000	....	....	—60.6	—48.5	....	—60	—62	—48.1	— 61.0†
9000	....	—48.0	—57.0	—44.6	—56.8	—51	—56	—43.4	— 54.5†
8000	....	—47.4	—51.0	—34.9	—48.7	—47	—48	—38.5	— 47.9†
7000	....	—38.4	—44.8	—31.7	—39.8	—38	—41	—33.3	— 39.6†
6000	....	—32.0	—37.5	—26.9	—34.6	—30	—34	—28.1	— 32.9†
5000	—20.8	—25.5	—32.3	—23.1	—27.0	—25	—26	—22.8	— 26.0†
4000	—15.0	—19.6	—28.0	—19.0	—20.7	—18	—21	—17.9	— 19.9†
3000	—12.9	—14.3	—19.5	—13.0	—15.4	—13	—15	—13.1	— 14.5†
2000	— 7.9	— 8.5	—15.8	— 9.6	— 9.9	— 9	— 8	— 7.8	— 9.0†
1000	— 3.2	— 3.7	— 8.3	— 3.8	5.0	— 4	— 4	— 3.9	— 4.3†
0000	0.0	0.0	0.0	0.0	0.0	0	0	0.0	0.0†

A = 49 ascensions not above 5,000 meters in manned balloons.
B = 12 trips upward and 5 downward, not above 10,000 meters, in manned balloons.
C = 9 ascensions of unmanned balloons above 10,000 meters.
D = Bigelow's compiled data, Tables 156, I. II., International Cloud Report.
E = Berson's mean results, Meteorologische Zeitschrift, Oct. 1901, p. 449.
F = Teisserenc de Bort's mean results, Meteorologische Zeitschrift, Oct. 1901, p. 449.
G = Hergesell's mean results. Meteorologische Zeitschrift. Oct. 1901, p. 449.
H = Bigelow's mean results, Tables 157, I. II., International Cloud Report.
I = The mean of E, F, G up to 10,000 meters, and a gradient of 9° per 1,000 meters from 11,000 to 16,000 meters.
* Hergesell's assumed gradient 9° per 1,000 meters.
† Mean of E, F, G.

Finally the same tables were employed to discuss the important problem of the difference between an adiabatic atmosphere and the one given by the upper strata observations, whereby a new method was illustrated, with results in Table 162. The value of this computation depends, of course, upon the data B, T, e, adopted for the upper atmosphere, as measured by the balloon and kite ascensions. It was especially necessary to have the temperatures at high levels, and for this purpose I collected such material as was available up to the end of the year 1896, when I began this compilation, and for that purpose employed the 102 balloon ascensions enumerated in Table 155, embracing all those then available for the United States, England, France, Germany, and Russia. I expressed myself cautiously regarding the result, page 750, holding the computation as preliminary to a fuller one which would become possible when accurate observations had been accumulated for the upper air temperatures, and I have therefore had an interest in examining the Berlin report of the German balloon ascensions.[2] In the first volume of this work is contained the data for each ascension, and in the Meteorologische Zeitschrift, October, 1901, page 449, H. Hergesell gives a summary of the resulting free air temperatures. I have extracted the observed temperatures from this report, interpolated them to each round 1,000-meter level, and computed the total temperature fall from the surface to the respective strata, with the result given in Table 21 and fig. 23. If the ascensions are divided into three sets, A, those reaching heights between the surface and 5,000 meters, B, those between the surface and 10,000 meters, and C, those between the surface and 16,000



A. and B. Berlin observations with manned balloons.
C. Berlin observations with unmanned balloons.
D. Bigelow's summary from all countries.
H. Bigelow's adopted mean result.
I. Berlin adopted mean result.

FIG. 23.—Total temperature fall from the surface to high levels by several systems.

[2] Wissenschaftliche Luftfahrten. Assmann und Berson. 3 Bänden. Berlin, 1899.

meters, we have the following remarkable data. Class *A* contains 49 ascensions of manned balloons, and gives a temperature fall of 20.8° at the 5,000-meter level; class *B* contains 12 upward and 5 downward trips of manned balloons and gives a fall of 25.5° at the 5,000-meter level, or 5° more than class *A*; class *C* contains 12 ascensions of unmanned balloons, with a fall of 32.3° at 5,000 meters, or 11.5° more than in class *A*, and 57° at 9,000 meters, or 9° more than in class *B*. This class shows also a fall of 60.6° at 10,000 meters and 60.4° at 16,000 meters. These widely different temperature falls by classes *A*, *B*, *C* may possibly be explained by those who are familiar with the circumstances, but the fact deserves attention; also the other fact that there is no temperature fall between 10,000 and 16,000 meters as observed in the Berlin unmanned balloon ascensions. In column *D* is given the result of my own compilation found by taking the mean of all the figures as they stand in Tables 156, I, II; and on fig. 21 the line *D* is seen to fall between *A* and *B* and to cross *C* at the height of 12,000 meters.

In his review of the Berlin ascensions H. Hergesell gave the Berson results as shown as in column *E*, the Teisserenc de Bort results as in column *F*, and his own results as in column *G*. He also stated the conclusion that above 10,000 meters the adiabatic rate of temperature fall in free air prevails, and this may be considered as 9.0° per 1,000 meters, as suggested by him. Column *I* is the mean value of *E*, *F*, *G*, up to 10,000 meters, and from that level to 16,000 the fall is calculated at 9.0° per 1,000 meters, these values being plotted on fig. 21. Finally, by taking the means of the data given in Tables 157, I, II, which was derived from Charts 78, 79, as constructed to determine the gradients for each month in the year, we have the data of column *H*, also plotted on fig. 21. It is seen that my adopted result, *H*, lies midway between *A* and *B*, and is a fair average of all the ascensions taken in the unmanned balloons, while the adopted Berlin result, *I*, is 45° lower at 16,000 meters, giving at that level a temperature of —115° approximately. There is a further consideration of importance to be noted in this connection. E. Rogovsky in his paper on the "Temperature and composition of the atmospheres of planets and the sun," Astrophysics, November, 1901, discusses the temperature of the interplanetary medium (according to Pouillet —142° C., Froelich —131° to —127°), and assumes it to be —142° C. A fair assumption regarding the efficient depth of the atmosphere makes it 64,000 meters or about 40 miles, and hence we have the following data:

Height of atmosphere.	Bigelow.		Berlin.	
	Temperature.	Necessary gradients.	Temperature.	Necessary gradients.
Meters.	° *C.*	° *C.*	° *C.*	° *C.*
64,000	—142		—142	
		—1.8		—0.9
16,000	— 55		—100	
		—4.4		—7.2
Surface	15		15	

If the temperature falls from 15° at the surface to —55° at 16,000 meters with a gradient of about —4.4° per 1,000 meters, then to reach —142° at 64,000 meters the gradient should on the average be —1.8°. It will be seen by my Charts 78 and 79, International Cloud Report, that I adopted an increasingly slower temperature fall with the height in the strata above 10,000 meters, in accordance with this general view. If the Berlin theory is assumed that a fall of 9.0° per 1,000 meters prevails above the 10,000-foot level, then it must somewhere rapidly decrease to a very small gradient in order not to diminish the exterpolated temperatures far below that value assigned by certain astrophysicists to the celestial medium at the earth's distance from the sun. In fact the gradient becomes one-tenth of the adiabatic rate, which was actually assumed. If the temperature —260° C. is that of the interplanetary medium, as supposed by other writers, these inferences must be modified accordingly.

From these two considerations, (1) that my temperature system includes the data of the highest balloon ascensions, and (2) that my gradients are in harmony with the requirements of astrophysics, I shall let my computations on the heat difference between the adiabatic and the actual atmosphere stand as they were given in my report. The accurate measurement of the temperatures in the highest strata is a very difficult process, and all efforts to secure reliable results deserve the hearty support of meteorological physicists. There are several problems whose solution depends upon the possession of such data in a satisfactory form.

THE FIRST NATIONAL METEOROLOGICAL CONGRESS OF MEXICO.[1]

By Prof. FRANK H. BIGELOW.

The report of the proceedings of the first Meteorological Congress of Mexico has been published and contains the acts and resolutions and papers presented during the sessions of November 1, 2, 3, 1900, held under the auspices of the Scientific Society "Antonio Alzate." The president was Señor D. Manuel Fernandez Leal, and there were about thirty members present at the sessions in an official capacity, The proceedings opened at 9:20 a. m., Thursday, November 1, 1900, with an address by the President, after which the papers to be read were presented. In the afternoon the session opened at 3:35, C. A. Gonzalez presiding, at which a discussion and the adoption of resolutions occurred, the purpose being to indicate the necessary steps in the organization of a national meteorological service for weather forecasts and climatology along recent modern lines, as laid down by the International Meteorological Congresses. Also a report was approved on the formation of a survey of the atmosphere by cloud observations, in three classes: (1) direction and motion of clouds by eye, (2) by nephoscopes, (3) by theodolites and photogrammeters.

On Friday, November 2, at 9:20 a. m., F. R. Rey presiding, papers were read by S. Diaz, L. G. Léo, M. Moreno y Anda, Señorita R. Sánchez Suárez, and J. M. Romero. At 4:30 p. m., D. M. Leal presiding, resolutions were passed as to the hours of observation, reduction of temperatures to the mean of 24 hourly observations, computation of the vapor tension, reduction of the barometer to zero temperature and to sea level, classification of clouds, the computation of the mean direction of the wind, and as to various special observations.

On Saturday, November 3, at 9 a. m., G. B. y Puga presiding, the reading of papers was continued by A. Prieto, Leal, and Olmedo. A discussion took place with the adoption of the following resolutions:

The first National Meteorological Congress expresses its desire that the Federal Government should provide for the organization of a meteorological service upon a basis analogous to that which exists in the United States; especially, will it be desirable to secure a modification of the existing services, taking account of the elements which actually exist, in conformity with the following principles: (1) That the Central Meteorological Observatory of Mexico be recognized as the central office of the national service; (2) that it be the center of all the scientific relations; (3) that the Federal Government equip this office for that work; (4) that the government establish and equip other observatories in suitable localities for cooperation with it; (5) that the state governments organize a network of stations in their own districts; (6) that a suitable telegraphic service be developed for meteorological messages; and (7) that a commission be organized to further the development of these plans.

At 4:30 p. m., J. de M. Tamborrel presiding, the discussion was continued, and resolutions were adopted concerning the

[1] Actas, resoluciones y memorias del primer Congreso Meteorológico Nacional, iniciado por la Sociedad Científica "Antonio Alzate," y celebrado en la ciudad de México los dias 1, 2 y 3 de Novimbre de 1900. Mexico. 1901. 272 pp.